configurations. Two bodies were chosen. The first was a glass cylinder approximately $\frac{1}{8}$ in. in diameter, and the second was a flat glass plate approximately $\frac{1}{2}$ in. wide. The models were mounted approximately $\frac{1}{2}$ in. above the orifice spanning the jet.

The free molecular jet effusion for a source temperature of 300°C is shown in Fig. 2a. The ambient pressure here was approximately 10^{-5} mm Hg, which was the limit of the present pumping system. The source pressure obtained from the known vapor pressure curve for Na was approximately 10^{-2} mm Hg. The number of molecules per unit time escaping the orifice was calculated to be approximately 10^{15} by means of Eq. (7). From Eqs. (8) and (9) one obtains a mean free path upstream of the orifice of 1.5 cm and a Knudsen number of 40. The number density inside the source may be calculated provided the vapor pressure of Na is known. The number density at the orifice was found to be about 10^{14} cm^{-3}, which is considerably higher than required for flow visualization by resonance scattering. At a distance of 4 cm from the orifice, the number density of the Na atoms was about 10^{10} cm^{-3}. In the present experiment, it was not possible to eliminate the reflections of the incident beam from the bell jar. These reflections appear as bright spots in the photograph.

Figure 2b is a photograph of a $\frac{1}{8}$-in.-diam glass cylinder in the jet. The source conditions are the same as those in the first photograph. The dark shadow running to the right from the cylinder is the light shadow cast by the cylinder in the beam of sodium light. The darker shadow above the model indicates the atomless wake cast by the model. The edges of the wake are straight lines that intersect at the orifice. This indicates that the particle path from the source is a straight line. The field of view in the photographs is limited by the width of the incident sodium light beam. However, during the course of the experiment the beam was moved to a distance of approximately 6 cm from the source. The jet was still visible at this distance where the number density is lower than 10^{10} cm^{-3}. It appears, therefore, that the resonance scattering technique may be used at number densities considerably less than 10^{10} cm^{-3}.

The free molecular flow over a plate $\frac{1}{2}$ in. wide is shown in Fig. 2c. The source conditions are the same as before. The characteristic straight line wake behind the model is again evident.

Several attempts were made to increase the source density sufficiently to observe the formation of a shock wave in front of the plate. A practical difficulty arose when this was attempted. To increase the density of Na in the source chamber, the temperature must be raised considerably. It was found that the condensation plate, which was at room temperature, did not condense the Na jet completely, and consequently the entire bell jar was fogged and the features of the jet were masked out. Another problem that arose with the present source design was that, at higher temperatures, some of the liquid Na passed through the orifice. When this happened, the bell jar fogged rapidly and masked out the jet.

Conclusions

The results of this experiment show that the resonance scattering technique may be used to obtain good photographs in the free molecular flow regime. The technique has not yet been extended to the continuum and transitional regimes, but in principle there should be no great difficulty in photographing these regimes. The only requirement for use of the technique is to obtain number densities of the order of 10^{10} cm^{-3}. If the number densities are too high, multiple scattering will complicate the interpretation of the results. For this reason, it appears that to photograph higher density flow fields it may be desirable to seed the working gas with sodium to obtain visualization. It is clear that the same technique can be used not only in the case of sodium but in principle for all gases.

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Heat Loss and Reaction Order Effects on Heat Release of Two-Stage Combustion Chambers

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HOMOGENEOUS chemical or "well-stirred" reactor theory has been applied in recent years to a number of engineering studies of chemical kinetics in high output combustion operations.[1–3] Although some authors mention the effects of heat loss[4–6] and reaction order[5,6] on the heat release of single stage combustion chambers, no one has presented a detailed theoretical analysis of such effects. Since current interest is directed towards multiple stage combustion chambers,[3,7] the results presented herein describe heat loss and reaction order effects on the specific heat release of a two-stage combustion chamber.

Following the approach of DeZubay,[7] the operation of a two-stage combustion chamber is described by a system of two single-stage homogeneous chemical reactors connected in series. The first stage receives a fresh charge of fuel and oxidant. Additional fresh oxidant and the partially burned products from the first stage are mixed and permitted to react further in the second stage. Complete details of the model and nomenclature used here are presented in Ref. 7. The first and second stages operate at stoichiometric and twice stoichiometric air-fuel ratios, respectively. Both stages have the same volume, inlet temperature, and inlet pressure. Propane and air are chosen as the fuel and oxidant.

Heat Loss Effects

For the determination of heat loss effects, the overall reaction order n is taken as 2.0, and the exponents of the concentration terms in the rate equation are assumed equal to 1.0. Heat loss is expressed as a percentage B of the heat released. For convenience, B of the first stage is assumed equal to B of the second stage ($B_1 = B_2$).[‡]

For the derivation of equations describing the two-stage system, the reader is referred to Ref. 7. To include heat loss effects, Eqs. (5) and (7) of Ref. 7 are modified to

$$y_1 = \frac{c(\varphi_1 + 1)(T_1 - T_0)}{H\psi_1(1 - B)} \tag{1}$$

$$y_2 = \frac{c[(\varphi_2 + 1)T_2 - (\varphi_2 - \varphi_1)T_0 - (\varphi_1 + 1)T_1]}{H\psi_2(1 - B)(1 - \psi_1 y_1)} \tag{2}$$

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[‡] Subscripts 1 and 2 refer to the first and second stage, respectively.

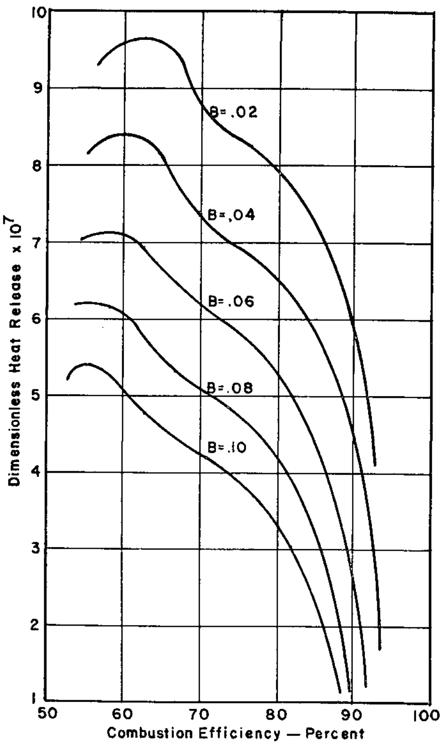

Fig. 1 Dimensionless heat release vs combustion efficiency for various percentages heat loss

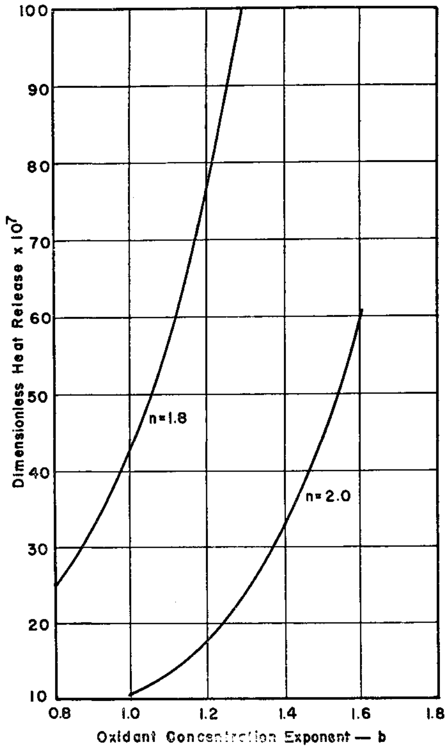

Fig. 2 Dimensionless heat release vs oxidant concentration exponent for various overall reaction orders

in the notation of Ref. 7. When Eqs. (1) and (2) are substituted in the derivation of Ref. 7, the equation relating the first-stage blowout temperature to other reactor parameters becomes

$$\frac{(n-b)}{(1-B)(T_f-T_0)-(T_1-T_0)}+\frac{1}{(T_1-T_0)}+\frac{n}{T_1}+$$

$$\frac{b\psi_1}{(1-B)(T_f-T_0)-\psi_1(T_1-T_0)}-\frac{E}{RT_1^2}=0 \quad (3)$$

Equation (3), along with the detailed trial and error procedure of Ref. 7, permits one to determine values of the (dimensionless) specific heat release as a function of (overall) combustion efficiency for various values of B.

Figure 1 shows the dimensionless heat release vs combustion efficiency for values of B in the range 2 to 10%. Heat losses in excess of 10% of the heat liberated are believed to be outside the range of practical interest.[3] The results of Fig. 1 compare most favorably with heat loss effects predicted by others.[5,6]

Reaction Order Effects

Flame thickness, burning velocity, and quenching distance methods yield values of the overall reaction order for hydrocarbon combustion in the range 1.2 to 2.0.[1] Reasons for the spread in values are beyond the scope of this discussion. Most investigators who use "well-stirred" reactor theory to predict combustion system performance assume (for mathematical convenience) the overall reaction order to be 1.8 or 2.0. They further assume, with the exception of Ref. 6, that the exponents of the concentration terms in the rate equation are equal to each other.

Using overall reaction orders n of 1.8 and 2.0 (which are the most common in propane-air studies) and varying the weight of fuel and oxidant concentrations, the equations and operating conditions previously described[7] yield Fig. 2. Here the dimensionless specific heat release is shown vs the oxidant concentration exponent b for two values of the overall reaction order n. The calculations of Fig. 2 include no heat loss effects.

As can be seen, a small change in reaction order or in the relative weight of the concentration terms causes a significant change in the predicted heat release values. This observation is contrary to that of Bittker and Brokaw,[8] who failed to account for dissociation effects.

Conclusions

The predicted heat release of a combustion system operating as a "well-stirred" two-stage reactor is very sensitive to heat losses and to small changes in the overall reaction order and/or changes in the concentration weighting.

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